

Decays of σ , κ , $a_0(980)$ and $f_0(980)$

D.V. Bugg^a

Queen Mary, University of London, London E1 4NS, UK

Received: 10 March 2006 /

Published online: 19 May 2006 – © Springer-Verlag / Società Italiana di Fisica 2006

Abstract. Ratios of coupling constants for these decays are compared with $q\bar{q}$ predictions and Jaffe's $q^2\bar{q}^2$ model. In both models, the predicted ratio $g^2(\kappa \rightarrow K\pi)/g^2(\sigma \rightarrow \pi\pi)$ is much too small. Also, for $q\bar{q}$, the predicted ratio $g^2(\kappa \rightarrow K\eta')/g^2(\kappa \rightarrow K\pi)$ is much larger than observed. Both models fail for $g^2(f_0 \rightarrow KK)/g^2(a_0 \rightarrow KK)$. This ratio requires that f_0 has a dominant KK component. It arises naturally because the f_0 pole lies very close to the KK threshold, giving its wave function a long KK tail.

PACS. 13.25.Gv; 14.40.Gx; 13.40.Hq

1 Introduction

There are conflicting opinions whether σ , κ , $a_0(980)$ and $f_0(980)$ are predominantly molecular states, $q\bar{q}$ or 4-quark. There are now extensive data for their coupling constants to pseudoscalars: (i) for σ and f_0 to $\pi\pi$, $\eta\eta$ and KK , (ii) for κ to $K\pi$, $K\eta$ and $K\eta'$, and (iii) for $a_0(980)$ to $\eta\pi$ and KK . The objective here is to compare all ratios of coupling constants with predictions for $q\bar{q}$ and $q^2\bar{q}^2$ states.

The σ pole has been known to generations of theorists, who extracted it from data on $\pi\pi$ elastic scattering, see the summary given by Markushin and Locher [1]. The E791 group then observed it as a peak in $D^+ \rightarrow \pi^+\pi^-\pi^+$ [2]. Higher statistics data from BES for $J/\Psi \rightarrow \omega\pi^+\pi^-$ now provide a better determination of the pole position $M - i\Gamma/2 = (541 \pm 39) - i(252 \pm 42)\text{MeV}$ [3]. If it is a $q\bar{q}$ state, one would expect a brother with $I = 1$ at a similar mass, whereas the $a_0(980)$ is over 400 MeV heavier.

Jaffe proposed that σ and its relatives are $q^2\bar{q}^2$ states [4]. His suggestion is that there is a pairing interaction forming S -wave diquarks in the flavour 3 configuration: ud , ds and us . Then 3 and $\bar{3}$ make a colourless nonet. The σ is the $I = 0$ member $u\bar{d}\bar{d}u$, the κ^+ is $u\bar{s}\bar{d}\bar{d}$, $a_0(980)$ is $s\bar{s}(u\bar{u} - \bar{d}\bar{d})/\sqrt{2}$ and $f_0(980)$ is $s\bar{s}(u\bar{u} + \bar{d}\bar{d})/\sqrt{2}$. This scheme neatly explains why a_0 and f_0 are nearly degenerate in mass and heavier than the σ by twice the mass of the s -quark. It also fits in neatly with the intermediate mass of the κ . From the latest combined analysis of E791, BES and LASS data, the κ pole is at $(750^{+30}_{-55}) - i(342 \pm 60)\text{MeV}$ [5].

There is support for Jaffe's scheme from recent Lattice QCD calculations of Okiharu et al. [6]. They find configurations at large radii consisting of a qq pair joined by a flux tube to a $\bar{q}\bar{q}$ pair. At small radii, they find two meson-meson pairs. The implication is that the massive "dressed"

$q^2\bar{q}^2$ configuration can decay by fission to two lighter pions at small separations.

Figure 1a shows diagrams for decays of $q\bar{q}$ states. Figure 1b shows fall-apart decays for four-quark states. In principle, four-quark combinations can make not only nonets but higher $SU(3)$ multiplets, which Jaffe discusses in detail. One can view his hypothesis as a final-state interaction which favours the nonet configuration.

An alternative possibility is that σ and its relatives are 'molecular' states created by long-range meson exchanges. There is a long history of proposals along these lines [7–10]. If one takes the K -matrix element in the s -channel from these Born terms, the unitarised amplitude $K/(1 - i\rho K)$ reproduces the observed $\pi\pi$ S -wave quite well up to 1 GeV and beyond. In all cases, the attraction from exchanges is barely sufficient to produce resonances. Indeed, for $a_0(980)$, the Jülich group, Janssen et al. [10] find only a virtual state. Likewise, unpublished calculations by Zou and myself find that there is not quite enough attraction from $K^*(890)$ and $\rho(770)$ exchange to generate $f_0(980)$ or $a_0(980)$. Because of this, widths predicted for a_0 and f_0 are particularly sensitive to meson coupling constants. As a result, no comparison will be made here with the 'molecular' hypothesis.

Although the amplitudes for $\pi\pi$ and $K\pi$ elastic scattering can be predicted adequately from meson exchanges, the nonet of ρ , ω , $K^*(890)$ and ϕ cannot be predicted in this way. Instead they appear as CDD poles [11]. This suggests that σ , κ , $a_0(980)$ and $f_0(980)$ are not regular $q\bar{q}$ states, although their formation may be related to short-range $q\bar{q}$ components, as in the approach of van Beveren et al. [12–14]. A new comparison with their model is in preparation.

Section 2 introduces some caveats. Both $f_0(980)$ and $a_0(980)$ lie close enough to the KK threshold that they must contain substantial KK components, resembling the

^a e-mail: D.Bugg@rl.ac.uk

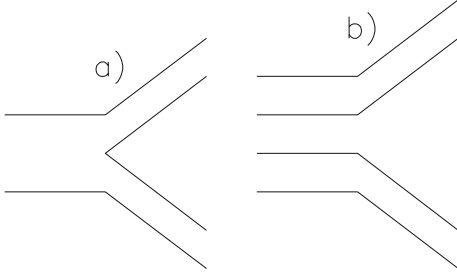


Fig. 1. Decays of a $q\bar{q}$, b $q^2\bar{q}^2$ states

long-range tail of the deuteron. It will be shown that the ratio of these long-range components is close to 2, and can account for the experimental ratio $g^2(f_0 \rightarrow KK)/g^2(a_0 \rightarrow KK)$. The experimental data also suggest mixing between σ and $f_0(980)$ in a mass range centred on the KK threshold.

Section 3 shows that both $q\bar{q}$ and $q^2\bar{q}^2$ schemes fail to account for several ratios of g^2 . Both schemes predict $g^2(f_0(980) \rightarrow KK)/g^2(a_0(980) \rightarrow KK)$ close to 1, in disagreement with the experimental value 2.15 ± 0.4 . In Jaffe's scheme, this problem may be remedied by taking $f_0(980)$ to be dominantly KK . There is, however, a residual problem in describing $\sigma \rightarrow KK$. The $q\bar{q}$ hypothesis fails to fit the ratio $g^2(f_0(980) \rightarrow KK)/g^2(a_0(980) \rightarrow KK)$ even when $f_0(980)$ is taken to be dominantly KK . Both schemes fail to account for the branching ratio $g^2(\kappa \rightarrow K\pi)/g^2(\sigma \rightarrow \pi\pi)$.

Section 4 points out the possible existence of a narrow state at the $\eta'\eta'$ threshold. This does not fit into Jaffe's model. The summary in Sect. 5 attempts to reach some conclusions.

From this point onwards, $a_0(980)$ and $f_0(980)$ will be abbreviated to a_0 and f_0 unless there is possible confusion with other states.

2 $f_0(980)$ must have a large KK component

At a mass just below the KK threshold, both f_0 and a_0 must have a long range tail due to small binding energy. Törnqvist [15] discusses this issue. His eqn. (15) gives a formula for the KK component in the wave function:

$$\psi = \frac{|q\bar{q}\rangle + \sum_i [-(\mathrm{d}/\mathrm{d}s) \operatorname{Re} \Pi_i(s)]^{1/2} |A_i B_i\rangle}{1 - \sum_i (\mathrm{d}/\mathrm{d}s) \operatorname{Re} \Pi_i(s)}, \quad (1)$$

where AB stands for molecular components KK , $\eta\eta$, $\pi\eta$, etc. The quantity Π is the propagator of the resonance and $\operatorname{Re} \Pi_{KK}(s) = g_{KK}^2 \sqrt{4m_K^2/s - 1}$ for $s < 4m_K^2$; there is a corresponding term for $\eta\eta$. [Törnqvist's equation is written in terms of $q\bar{q}$, but could equally well be reformulated in terms of 4-quark states]. His formula is easily evaluated to find the $K\bar{K}$ components in a_0 and f_0 as functions of s . At the $K\bar{K}$ threshold, the binding energy goes to zero and the KK wave function extends to infinity, so the KK fraction $\rightarrow 1$. Using BES parameters [16], the $f_0(980)$ has

a second sheet pole at $(998 \pm 4) - i(17 \pm 4)\text{MeV}$, very close to the KK threshold; there is a distant third sheet pole at $(851 \pm 28) - i(418 \pm 72)\text{MeV}$. The dominance of the narrow second sheet pole is used by Baru et al. [17] to argue that $f_0(980)$ is mostly a KK bound state pole. The $a_0(980)$ with parameters derived in [18] has a second sheet pole at $1032 - i85\text{MeV}$ and a third sheet pole at $968 - i245\text{MeV}$. This is closer to a conventional resonance and further from the KK threshold.

Results from Törnqvist's formula are shown in Fig. 2 by the dotted curves. This figure also shows line-shapes as the full curves. If the KK component behaves as an inert cloud for radii > 0.6 fm, the mean KK fraction integrated over the line-shape is $\geq 70\%$ for f_0 and $\sim 35\%$ for a_0 .

2.1 Further caveats

It is first necessary to explain the view adopted here for the broad σ . It is a very curious resonance with unusual features. Achasov and Shestakov were amongst the first to clarify the relation with chiral symmetry within the framework of the linear σ model [19]. They pointed out that it cannot be fitted adequately by a simple Breit-Wigner resonance with a large constant width. It has a pole at ~ 540 MeV, but the observed $\pi\pi$ elastic phase shift goes through 90° at ~ 1 GeV. How are these two facts reconciled?

The clue is that the width is strongly s -dependent, with a zero at the Adler point $s \simeq 0.5m_\pi^2$, just below threshold. Experiments on $\pi\pi$ elastic scattering are done at real values of s . In finding the pole, it is necessary to extrapolate the measured amplitude off the real s -axis. The Breit-Wigner amplitude fitted to the data has a width of the form $\Gamma = A(s)(s - s_A)\rho_{\pi\pi}(s)$, where ρ is the usual Lorentz invariant phase space $\sqrt{1 - 4m_\pi^2/s}$; $A(s)$ is a slowly varying function of s . The phase shift goes to 0 at the $\pi\pi$ threshold. However, the pole lies at $s_0 = 0.23 - i0.27\text{GeV}^2$. In the extrapolation to the pole, the factor $(s - s_A)\rho_{\pi\pi}(s)$ develops a large phase rotation of $\sim 55^\circ$ near the pole. Oller drew attention to this earlier [20]. The result is that the pole is approximately 55° ahead of the phase of the amplitude along the real axis; there is a further small phase variation arising from the slowly varying function $A(s)$, but it is only a few degrees in practice. Qualitatively, the broad $\pi\pi$ amplitude measured on the real s -axis may be viewed as a long tail of the pole buried deep in the complex s -plane. Its phase reaches 90° for real $s \sim 1$ GeV². In production data, one sees a peak in the $\pi\pi$ intensity at ~ 500 MeV, but that peak is hidden in $\pi\pi$ elastic scattering by the Alder zero in that process [21]. Figure 2c of that reference shows a graph of the mass at which the phase passes 90° for complex s .

The broad component of the $\pi\pi$ S -wave continues through the mass range 1 to 2 GeV. It stretches the imagination to interpret it as the tail of the σ pole. It may therefore have a further origin in that mass range. For example, Anisovich et al. [22] argue that it should be interpreted there as a broad glueball.

For the κ , the situation is even more extreme. The phase rotation between the pole and the real s -axis is

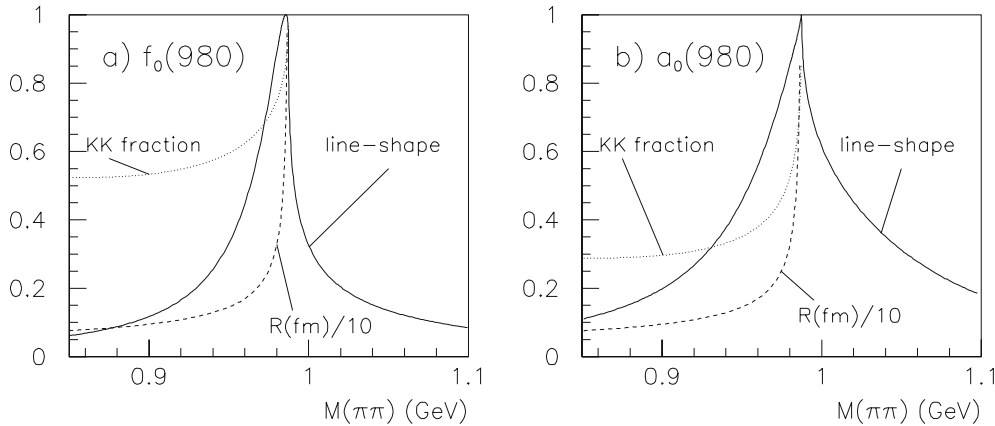


Fig. 2. Line shapes of $f_0(980)$ and $a_0(980)$, *full curves*; the KK fraction in the wave function, *dotted curves*; $0.1 \times$ RMS radius, *dashed curves*

$\sim 85^\circ$. The long tail of this pole is fitted to $K\pi$ elastic phase shifts determined in the LASS experiment of Aston et al. [23]. Because of the very large phase rotation, these phase shifts do not quite reach 90° over the mass range where they have been measured.

The view being examined here is a narrow one, that the amplitude for the so-called σ in the vicinity of the KK and $\eta\eta$ thresholds may be expressed in terms of just two orthogonal states $f_0(980)$ and σ . Likewise, the amplitude for the κ may be expressed in terms of just two orthogonal states κ and $K_0(1430)$. This could be an oversimplification. The objective is to see where this view leads.

Resonances have a finite spatial extent. The $\pi\pi$ S -wave amplitude is known up to 1.9 GeV. One can take the Fourier transform of the observed s -dependence to determine the radius of interaction. The result is a rather small RMS radius of 0.45 fm [21]. This is quite enough to produce a large form factor between the mass of the σ pole and 1 GeV, and likewise between the mass of the κ and the $K\eta$ and $K\eta'$ thresholds. One should remain alert to the fact that coupling parameters are likely to be s -dependent. It is therefore not realistic to use ratios like $g^2(a_0 \rightarrow \eta\pi)/g^2(\sigma \rightarrow \pi\pi)$ for quantitative purposes, because the poles are too far apart.

Data on ϕ radiative decays are analysed in an accompanying paper [24] and provide a precise measurement of the ratio $g^2(\sigma \rightarrow KK)/g^2(\sigma \rightarrow \pi\pi)$ from interference with $f_0(980)$. This will be taken as a reliable number at the KK threshold. Data on $\pi\pi \rightarrow KK$ are also analysed in [24]. These data are fitted over a range of masses up to 1.9 GeV; they appear to confirm the result from KLOE data within a somewhat larger error.

There is a further caveat which is rarely discussed. For an isolated σ produced without $f_0(980)$, there is a multiple scattering series for $\sigma \rightarrow \pi\pi$ and KK . The $f_0(980)$ has its own multiple scattering series. In $\pi\pi$ elastic scattering, both the broad σ and $f_0(980)$ appear strongly at 1 GeV in Cern–Munich data [25]. The multiple scattering series then contains additional terms of the form $\sigma \rightarrow \pi\pi \rightarrow f_0$ and vice versa. These cross-terms are likely to lead to dynamical mixing of $f_0(980)$ and σ unless the overlap of their wave function happens to be zero. However, this mixing can vary from process to process, depending on how much of each is produced in the formation reaction. For example,

in $J/\Psi \rightarrow \omega\pi\pi$, the σ is produced strongly, but there is little or no $f_0(980)$ [3]. In $J/\Psi \rightarrow \phi\pi^+\pi^-$, the $f_0(980)$ is produced strongly with a small σ amplitude accompanying it [16].

In elastic scattering, both σ and $f_0(980)$ are produced strongly. The analysis of data in the accompanying paper [24] shows that a substantial $\sigma \rightarrow KK$ component is needed near the KK threshold with $g^2(\sigma \rightarrow KK)/g^2(\sigma \rightarrow \pi\pi) = 0.6 \pm 0.1$. However, it appears to be somewhat localised near the KK threshold. KLOE data on $\phi \rightarrow \gamma\pi^0\pi^0$ will not tolerate a $\sigma \rightarrow KK$ component with the large width of the σ ; this amplitude must be attenuated strongly below ~ 800 MeV. It may be fitted using a rather strong form factor $\exp(-\alpha|k|^2)$ where k is KK centre-of-mass momentum.

Above the KK threshold, data on $\pi\pi \rightarrow KK$ again require a rather strong form factor to fit the observed strong decrease in the cross section from 1 to 1.8 GeV. The result is a broad peak in the coupling to KK over a mass range roughly 800 to 1300 MeV. A straightforward possibility is that there is mixing between $f_0(980)$ and σ , peaking there. For this reason, the analysis in the next Section will focus on ratios of g^2 only close to KK and $\eta\eta$ thresholds.

2.2 Systematic errors for coupling constants

Flatté formulae have been used in fitting a_0 and f_0 , but ignoring coupling of $a_0(980)$ to $\eta'\pi$ and $f_0(980)$ to $\eta\eta$. Tests adding these couplings suggest that effects are small compared with errors assigned by the experimental groups. The BES data have been refitted allowing the $\eta\eta$ coupling explicitly, but the data suggest no coupling to this channel. The main source of systematic error for the f_0 is the effect of possible mixing with σ . This mixing depends on unknown wave functions. The coupling constant for $\sigma \rightarrow KK$ is particularly sensitive to this mixing, but errors assigned in [24] are intended to cover the range of possible form factors fitted to both σ and f_0 .

The σ pole was determined by the BES collaboration [3] fitting several different Breit–Wigner forms. The quoted systematic errors cover this range of possibilities. Some authors have raised the possibility of unknown ‘non-resonant backgrounds’ in the σ . However, without educated guesses about such possible backgrounds, there is

no limit to the possible range of parameters which can be fitted. The approach adopted here is to fit a simple empirical s -dependent width including the Adler zero; errors for the fitted parameters cover the likely range of possibilities. The same approach is adopted for the κ . There, the main problem in fitting parameters is unknown mixing with $K_0(1430)$. However, the latest refit to LASS, BES and E791 data [5] arrives at a consistent picture from the three sets of data; the range of parameters fitting all three sets of data will be used to cover possible systematic errors. There is a small systematic discrepancy with LASS data around 1.2 GeV, and its effect on possible coupling to $K\eta$ will be discussed in Sect. 3.3.

Parameters of σ and κ are completely insensitive to precise locations of the Adler zeros.

3 Comparison of g^2 with $q\bar{q}$ and Jaffe's model

It is necessary first to discuss the selection of data used for this comparison. The σ pole will be taken from the high statistics data of BES, where there is a clearly visible peak with a well defined mass and width [3]. The κ pole will be taken from the combined analysis of E791 and BES production data and LASS phase shifts [5]. The $f_0(980)$ appears as a strong peak in BES data for $J/\Psi \rightarrow \phi\pi^+\pi^-$ and ϕK^+K^- [16]. Its width is precisely determined by the $\pi\pi$ peak because of the very good mass resolution. The signal is clearly visible near threshold in the KK channel. The ratio of events in this peak to that in the $\pi\pi$ peak determines the branching ratio $g^2(KK)/g^2(\pi\pi)$ accurately.

The $a_0(980)$ is subjected to detailed scrutiny in an accompanying paper [24] which compares a fit to Kloe data on $\phi \rightarrow \gamma\eta\pi^0$ with an earlier determination from Crystal Barrel data [18]. There is agreement within errors, and values of g^2 are taken from the combined analysis. Most other experimental determinations quoted by the Particle Data group are fitted to a Breit–Wigner resonance of constant width, an assumption far from reality.

The comparison of g^2 made here is motivated by a similar comparison for well known $q\bar{q}$ states such as $\rho(770)$ and $K^*(890)$. After allowing for effects due to identical particles (discussed in detail in the next section), the prediction for $g^2(K^*(890) \rightarrow K\pi)/g^2(\rho \rightarrow \pi\pi)$ is 3/4. This agrees well with experiment if one allows a P -wave Blatt–Weisskopf centrifugal barrier for both decays with a reasonable radius of 0.5 fm. For σ and its relatives, no centrifugal barrier is involved between mesons, so a comparison of g^2 should be a meaningful test of the models.

3.1 Formulae

Formulae for coupling of σ , κ , a_0 and f_0 to $q\bar{q}$ have been given by Anisovich, Anisovich and Sarantsev [22]. Corresponding formulae for $q^2\bar{q}^2$ are given by Jaffe in Table 7 of his publication [4]. However there is an important subtlety concerned with these formulae for identical particles $\pi\pi$ and $\eta\eta$.

Consider the Breit–Wigner amplitude a for a process involving non-identical particles: $K^+K^- \rightarrow a_0 \rightarrow \eta\pi$,

$$a = \frac{g_{K^+K^-}g_{\eta\pi}}{m^2 - s - i(g_{\eta\pi}^2\rho_{\eta\pi} + g_{KK}^2\rho_{KK})}. \quad (2)$$

The integrated cross section involves an integral 4π over the solid angle. For $K\bar{K} \rightarrow f_0 \rightarrow \pi^0\pi^0$, isospin Clebsch–Gordan coefficients combining two isospins 1 to $I=0$ lead to a final state $(\pi^+\pi^- - \pi^0\pi^0 + \pi^-\pi^+)/\sqrt{3}$. The $\pi^+\pi^-$ cross section may be determined by counting π^+ over the whole solid angle. At a particular angle θ , there are two amplitudes $\pi^+(\theta)\pi^-(\theta+\pi)$ and $\pi^-(\pi+\theta)\pi^+(\theta)$ which add coherently. The integrated intensity over angles is $(4/3)4\pi$. For $\pi^0\pi^0$, there are again contributions $\pi^0(\theta)\pi^0(\pi+\theta)$ and $\pi^0(\pi+\theta)\pi^0(\theta)$, but the angular integration should now be done over only one hemisphere, to avoid counting both π^0 from a single event. The result is $(4/3)2\pi$. The total $\pi\pi$ integral is $(6/3)4\pi$. So the identity of the pions leads to a doubling of the $\pi\pi$ branching ratio and $g_{\pi\pi}^2$, and likewise for $\eta\eta$.

The hypothesis to be tested here is that all members of a $q\bar{q}$ nonet have the same coupling constant g (apart from effects of identical particles and Clebsch–Gordan coefficients). Anisovich and Sarantsev include the factor 2 for identical particles explicitly into branching ratios for $\pi\pi$ and $\eta\eta$. That convention will be followed here. However, Jaffe gives formulae for amplitudes without the identity factor and leaves the user to put it in.

For $q\bar{q}$ states, the following linear combinations will be used:

$$\sigma = n\bar{n} \cos \phi + s\bar{s} \sin \phi \quad (3)$$

$$f_0 = -n\bar{n} \sin \phi + s\bar{s} \cos \phi, \quad (4)$$

where $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$. Diagrams of Fig. 1a for decay of $I=0$ $q\bar{q}$ states lead to a final state

$$A = [u(\bar{u}u + \bar{d}d + \sqrt{\lambda}s\bar{s})\bar{u} + d(\bar{u}u + \bar{d}d + \sqrt{\lambda}s\bar{s})\bar{d}] \frac{\cos \phi}{\sqrt{2}} + s[\bar{u}u + \bar{d}d + \sqrt{\lambda}s\bar{s}] \sin \phi. \quad (5)$$

The factor $\sqrt{\lambda}$ is introduced by Anisovich and Sarantsev to allow for possible differences between $\bar{n}n$ and $\bar{s}s$. There is a one-to-one correspondence between each term in this series and the possible diagrams of Fig. 1a.

The η and η' will be written as

$$\eta = n\bar{n} \cos \theta_P - s\bar{s} \sin \theta_P \quad (6)$$

$$\eta' = n\bar{n} \sin \theta_P + s\bar{s} \cos \theta_P \quad (7)$$

$$\eta_0 = \eta \cos \theta_P + \eta' \sin \theta_P \quad (8)$$

$$\eta_s = -\eta \sin \theta_P + \eta' \cos \theta_P, \quad (9)$$

where θ_P is the pseudoscalar mixing angle; the value $\sin \theta_P = 0.608 \pm 0.025$ will be used [26]. A straightforward

expansion of (5) gives

$$A = [\eta\eta \cos^2 \theta_P + \pi^0\pi^0 - \pi^-\pi^+ - \pi^-\pi^+ + \sqrt{\lambda}(K^0\bar{K}^0 - K^-K^+)] \frac{\cos \phi}{\sqrt{2}} + [K^0\bar{K}^0 - K^-K^+ + \sqrt{\lambda}\eta\eta \sin^2 \theta_P] \sin \phi; \quad (10)$$

contributions from $\eta\eta'$ and $\eta\eta$ have been omitted for simplicity.

The a_0^+ leads to a final state

$$F(a_0) = u(\bar{u}u + \bar{d}d + \sqrt{\lambda}\bar{s}s)\bar{d}; \quad (11)$$

after using G parity to eliminate $\pi\pi$ final states (or alternatively the Pauli principle), the surviving amplitude is

$$F(a_0) = \frac{1}{\sqrt{2}}(\eta_0\pi^+ + \pi^+\eta_0) + \sqrt{\lambda}K^+\bar{K}^0. \quad (12)$$

For the κ^+ ,

$$F(\kappa^+) = u(\bar{u}u + \bar{d}d + \sqrt{\lambda}\bar{s}s)\bar{s} \rightarrow \frac{1}{\sqrt{2}}(\eta_0 - \pi^0)K^+ + \pi^+K^0 + \sqrt{\lambda}\eta_s K^+. \quad (13)$$

(14)

Resulting $q\bar{q}$ branching ratios (integrated over charge states) are shown in column 2 of Table 1.

Jaffe's model requires a non-strange $I = 0$ component N which may be written

$$N = (1/2)(u\bar{d}\bar{d}\bar{u} + d\bar{u}\bar{d}\bar{d} + u\bar{u}\bar{d}\bar{d} + d\bar{d}\bar{u}\bar{u}) \quad (15)$$

$$= (1/2)(\eta_0\eta_0 + \pi^0\pi^0 - \pi^-\pi^+ - \pi^+\pi^-). \quad (16)$$

There is an orthogonal state with hidden strangeness

$$S = (1/2)(u\bar{u}s\bar{s} + d\bar{d}s\bar{s} + u\bar{s}s\bar{u} + d\bar{s}s\bar{d}) \quad (17)$$

$$= (1/2)(\sqrt{2}\eta_0\eta_s - K^+K^- + K^0\bar{K}^0). \quad (18)$$

Further states are

$$a_0^+ = (1/\sqrt{2})(u\bar{d}s\bar{s} + u\bar{s}s\bar{d}) \quad (19)$$

$$= (1/\sqrt{2})(\pi^+\eta_s + K^+\bar{K}^0), \quad (20)$$

$$\kappa^+ = (1/\sqrt{2})(u\bar{s}d\bar{d} + d\bar{s}u\bar{d}) \quad (21)$$

$$= (1/2)[K^+(\eta_0 + \pi^0) + \sqrt{2}K^0\pi^+]. \quad (22)$$

The quantities N and S replace $n\bar{n}$ and $s\bar{s}$ in (3) and (4). The third column of Table 1 shows branching ratios for Jaffe's model.

3.2 Conclusions from $f_0(980)$, $a_0(980)$ and σ

The parameter λ of Anisovich and Sarantsev was preserved in Table 1 for reference purposes; however, it does not systematically improve agreement with experiment, so it will

Table 1. Ratios of g^2 predicted by $q\bar{q}$ and $q^2\bar{q}^2$ models; $c = \cos \theta_P$, $s = \sin \theta_P$

Ratio of g^2	$q\bar{q}$	$q^2\bar{q}^2$
$(\kappa \rightarrow K\pi)/(\sigma \rightarrow \pi\pi)$	$1/(2\cos^2 \phi)$	$1/(2\cos^2 \phi)$
$(\kappa \rightarrow K\eta)/(\kappa \rightarrow K\pi)$	$(c - \sqrt{2}\lambda s)^2/3$	$c^2/3$
$(\kappa \rightarrow K\eta')/(\kappa \rightarrow K\pi)$	$(s + \sqrt{2}\lambda c)^2/3$	$s^2/3$
$(a_0 \rightarrow \pi\eta)/(a_0 \rightarrow KK)$	$2c^2$	s^2
$(a_0 \rightarrow \pi\eta')/(a_0 \rightarrow KK)$	$2s^2$	c^2
$(\sigma \rightarrow \eta\eta)/(\sigma \rightarrow \pi\pi)$	$(c^2 + \sqrt{2}\lambda s^2 \tan \phi)^2/3$	$(c^2 - \sqrt{2}cs \tan \phi)^2/3$
$(\sigma \rightarrow KK)/(\sigma \rightarrow \pi\pi)$	$(\sqrt{\lambda} + \sqrt{2} \tan \phi)^2/3$	$(1/3) \tan^2 \phi$
$(f_0 \rightarrow \eta\eta)/(f_0 \rightarrow \pi\pi)$	$(c^2 - \sqrt{2}\lambda s^2 \cot \phi)^2/3$	$(c^2 + \sqrt{2}cs \cot \phi)^2/3$
$(f_0 \rightarrow KK)/(f_0 \rightarrow \pi\pi)$	$(\sqrt{\lambda} - \sqrt{2} \cot \phi)^2/3$	$(1/3) \cot^2 \phi$
$(f_0 \rightarrow KK)/(a_0 \rightarrow KK)$	$(\sin \phi - \sqrt{2/\lambda} \cos \phi)^2$	$\cos^2 \phi$

Table 2. Ratios of g^2 for $q\bar{q}$ and $q^2\bar{q}^2$ models predicted from $g^2(f_0 \rightarrow KK)/g^2(f_0 \rightarrow \pi\pi)$, compared with experimental values from [24]; alternative solutions are with ϕ positive (first solution) or negative (second)

Ratio of g^2	$q\bar{q}$	$q^2\bar{q}^2$	Expt
$(f_0 \rightarrow \eta\eta)/(f_0 \rightarrow \pi\pi)$	0.37 ± 0.14 or 0.83 ± 0.09	3.11 ± 0.08 or 1.07 ± 0.18	< 0.33
$(f_0 \rightarrow KK)/(a_0 \rightarrow KK)$	1.11 ± 0.04 or 2.96 ± 0.03	0.93 ± 0.01	2.15 ± 0.4
$(\sigma \rightarrow KK)/(\sigma \rightarrow \pi\pi)$	0.69 ± 0.02 or 0.02 ± 0.01	0.03 ± 0.01	0.6 ± 0.1
$(\sigma \rightarrow \eta\eta)/(\sigma \rightarrow \pi\pi)$	0.21 ± 0.01 or 0.04 ± 0.01	0.06 ± 0.02 or 0.23 ± 0.02	0.20 ± 0.04

be set to 1. The initial objective is to show that the data for these decays are inconsistent with either $q\bar{q}$ or $q^2\bar{q}^2$ for $f_0(980)$.

Let us start from the ratio $g^2(f_0 \rightarrow KK)/g^2(f_0 \rightarrow \pi\pi) = 4.21 \pm 0.46$, which is well determined from recent BES data on $J/\psi \rightarrow \phi\pi^+\pi^-$ and ϕK^+K^- [16]; statistical and systematic errors have been combined in quadrature. These data lead to two possibilities for the mixing angle ϕ . For $q\bar{q}$, they are $(17.3 \pm 0.7)^\circ$ or $(-29.0 \pm 2.0)^\circ$. The errors cover purely experimental errors for the ratio $g(f_0 \rightarrow KK)/g^2(f_0 \rightarrow \pi\pi)$; in Table 2, errors from this source are propagated and added in quadrature with errors from the pseudoscalar mixing angle θ_P . The first solution, $\phi = +17.2^\circ$ agrees with experiment for three ratios, but fails for $g^2(f_0 \rightarrow KK)/g^2(a_0 \rightarrow KK)$. The second solution, $\phi = -29.0^\circ$ fails for three ratios.

The $q^2\bar{q}^2$ scheme leads to two solutions with $\phi = \pm(15.7 \pm 0.9)^\circ$. Neither solution agrees with all experimental ratios. The branching ratio of $f_0 \rightarrow \eta\eta$ is far above the experimental limit and the branching ratio for $\sigma \rightarrow KK$ is far below experiment.

In view of the prediction from Sect. 2 that $f_0(980)$ should contain a large KK component, we immediately turn to the case where $f_0(980)$ is pure KK :

$$\sigma = n\bar{n} \cos \phi + KK \sin \phi \quad (23)$$

$$f_0 = -n\bar{n} \sin \phi + KK \cos \phi \quad (24)$$

$$S = KK. \quad (25)$$

Results are shown in Table 3. For $q\bar{q}$, the only change to (10) is the disappearance of the term $\eta\eta \sin^2 \theta_P$. There is therefore no change to values of ϕ , and entries 2 and 3 remain unchanged. Entry 1 is marginally improved and entry 4 is slightly worse. There is no improvement in the ratio $g^2(f_0 \rightarrow KK)/g^2(a_0 \rightarrow KK)$.

For Jaffe's model, S of (18) is replaced by $(1/\sqrt{2})(K^0\bar{K}^0 - K^+K^-)$. The value of ϕ changes to $\pm(21.7 \pm 1.2)^\circ$; there is an improvement in the ratio $g^2(f_0 \rightarrow KK)/g^2(a_0 \rightarrow KK)$ to a value within one standard de-

variation of experiment. However, entry 3 is still far from experiment.

One can try to improve the agreement for the $q^2\bar{q}^2$ scenario by including a small amount of the hidden strange component S of Jaffe's model, using

$$\sigma = N \cos \phi + \frac{\alpha S + KK}{\sqrt{1 + \alpha^2}} \sin \phi \quad (26)$$

$$f_0 = -N \sin \phi + \frac{\alpha S + KK}{\sqrt{1 + \alpha^2}} \cos \phi. \quad (27)$$

However, it turns out that there is no solution which gives agreement with both $f_0 \rightarrow \pi\pi$ and $\sigma \rightarrow KK$. The best that can be achieved is to increase $g^2(\sigma \rightarrow KK)/g^2(\sigma \rightarrow \pi\pi)$ to 0.29, still a factor 2 smaller than experiment.

3.3 Results for the κ

Table 4 shows predictions for σ , κ and a_0 . For the first entry, the predicted ratio is almost the same for $q\bar{q}$ and $q^2\bar{q}^2$; the best values of ϕ are chosen from Table 3. At the position of the κ pole, $|\rho_{K\pi}| = 0.821$ and at the σ pole $|\rho_{\pi\pi}| = 0.936$. Using the width observed for the σ pole by BES, 504 ± 84 MeV, both $q\bar{q}$ and $q^2\bar{q}^2$ predict a κ width of 236 ± 39 MeV. Such a narrow κ would be extremely conspicuous; it is completely ruled out by the data, which require a width roughly a factor 3 larger [5]. Nonetheless, the experimental ratio $g^2(\kappa \rightarrow K\pi)/g^2(\sigma \rightarrow \pi\pi)$ quoted in Table 4 requires some explanation. The first value 2.14 ± 0.28 is obtained from the conventional expression $M\Gamma/|\rho|$ at the pole. However, it is debatable what effect the Adler zero has on the width. Experimentally, the width is parametrised as $A(s)(s - s_A)$, where $A(s)$ is a slowly varying exponential factor preventing the width from increasing continuously with s . The σ pole lies closer to its Adler zero than the κ pole. The Adler zero might therefore suppress the width. An extreme view is to factor the term $(s - s_A)$ out of the width and examine the ratio of

Table 3. Ratios of g^2 for $q\bar{q}$ and $q^2\bar{q}^2$ models predicted with f_0 pure KK

Ratio of g^2	$q\bar{q}$	$q^2\bar{q}^2$	Experiment
$(f_0 \rightarrow \eta\eta)/(f_0 \rightarrow \pi\pi)$	0.13 ± 0.01	0.13 ± 0.01	< 0.33
$(f_0 \rightarrow KK)/(a_0 \rightarrow KK)$	1.11 ± 0.04 or 2.96 ± 0.03	1.73 ± 0.03	2.15 ± 0.4
$(\sigma \rightarrow KK)/(\sigma \rightarrow \pi\pi)$	0.69 ± 0.02 or 0.02 ± 0.01	0.02 ± 0.02	0.6 ± 0.1
$(\sigma \rightarrow \eta\eta)/(\sigma \rightarrow \pi\pi)$	0.13 ± 0.01	0.13 ± 0.01	0.20 ± 0.04

Table 4. Ratios of g^2 predicted by $q\bar{q}$ and $q^2\bar{q}^2$ models and experimental values

Ratio of g^2	$q\bar{q}$	$q^2\bar{q}^2$	Expt
$(\kappa \rightarrow K\pi)/(\sigma \rightarrow \pi\pi)$	0.55	0.58	2.14 ± 0.28 to 1.35 ± 0.10 [3, 5]
$(\kappa \rightarrow K\eta)/(\kappa \rightarrow K\pi)$	0.004 ± 0.005	0.20 ± 0.01	0.06 ± 0.02 [5]
$(\kappa \rightarrow K\eta')/(\kappa \rightarrow K\pi)$	1.00 ± 0.01	0.13 ± 0.01	0.29 ± 0.29 [5]
$(a_0 \rightarrow \pi\eta)/(a_0 \rightarrow KK)$	1.21 ± 0.06	0.40 ± 0.03	0.75 ± 0.11 [25]
$(a_0 \rightarrow \pi\eta')/(a_0 \rightarrow KK)$	0.79 ± 0.06	0.60 ± 0.03	—

$A(s)$ at the pole. This gives the second result of Table 2, 1.35 ± 0.10 . Incidentally, the small error arises from a cancellation between correlations involved in finding the pole position.

Consider next decays $\kappa \rightarrow K\eta'$. For $q\bar{q}$, entry 3 of Table 4 shows that the predicted ratio $g^2(\kappa \rightarrow K\eta')/g^2(\kappa \rightarrow K\pi)$ is large and ~ 2.5 standard deviations away from experiment. It is possible that the $K\pi$ signal may be attenuated by a form factor, but this would make the disagreement worse. If the broad κ signal were to couple strongly to $K\eta'$, one should see a strong dispersive effect in the vicinity of this threshold. There is no sign of any such effect in the data. It therefore appears that there is a discrepancy with the $q\bar{q}$ hypothesis.

Entry 2 shows predictions for $K\eta$. Both are small. Neither model makes an accurate prediction, though they both give small numbers like experiment. The $q^2\bar{q}^2$ model does not fare so well. However, a warning is that [5] points out that the fit to LASS data is not perfect around 1.2 GeV, fairly close to the $K\eta$ threshold. This problem may well arise because the s -dependence being fitted presently to the data is the simplest possible and may be oversimplified; the error quoted for the experimental value is purely statistical and does not allow for possible systematic error. The simple fact is that there is no evidence for structure in the broad $\kappa \rightarrow K\pi$ at the $K\eta$ threshold. It would be valuable to have data directly for the $K\eta$ channel.

The fourth entry of Table 5 compares $g^2(a_0 \rightarrow \eta\pi)$ with $g^2(a_0 \rightarrow KK)$. The experimental ratio is rather well known from the combination of Crystal Barrel data and KLOE data [24]. It is over three standard deviations larger than predicted by Jaffe's model. The $q\bar{q}$ prediction is higher than experiment. However, the momentum available in $\eta\pi$ decays is 325 MeV/c and a form factor with an RMS radius of 0.75 fm could bring the $q\bar{q}$ prediction into agreement with experiment; for the $q^2\bar{q}^2$ case, such a form factor would make matters worse. From Sect. 2, the $a_0(980)$ must contain a KK component of $\sim 35\%$. This is neither small nor large. It is possible that a more refined model allowing for this 35% KK component might change the level of agreement with the $q^2\bar{q}^2$ hypothesis, but it is not presently clear how to construct such a model. The present conclusion is that $q\bar{q}$ gives better agreement with experiment.

Table 4 shows predictions for $a_0 \rightarrow \pi\eta'$. Presently there are no data for this ratio. Such data are important to complete the picture.

3.4 Discussion

Neither $q\bar{q}$ nor Jaffe's model gives reasonable agreement with experiment. The failure to predict the ratio $g^2(f_0 \rightarrow KK)/g^2(a_0 \rightarrow KK)$ may reasonably be attributed to the fact that f_0 has a dominant KK cloud. With this correction, Jaffe's model predicts a ratio within 1σ of experiment. However, $q\bar{q}$ still fails to predict this ratio. This is because (12) predicts $g^2(a_0 \rightarrow KK)$ a factor 2 larger than (20) of Jaffe's model.

However, the critical point where both $q\bar{q}$ and Jaffe's model fail seriously is the prediction $g^2(\kappa \rightarrow K\pi)/g^2(\sigma \rightarrow$

$\pi\pi) = 1/(2\cos^2\phi)$. This is in complete contradiction with experiment.

An alternative scenario is that σ , κ , a_0 and f_0 are driven by meson exchanges [7–10]. These calculations show that $\pi\pi$ and $K\pi$ phase shifts may be reproduced by taking Born terms from the meson exchanges and unitarising the amplitude using the K-matrix. The calculations provide a valuable clue. All these resonances are only just bound. Coupling constants of mesons need to be adjusted (within their errors) to reproduce phase shifts for $\pi\pi$ and $K\pi$ and resonance masses and widths for f_0 and a_0 . As coupling constants increase, phase shifts vary more rapidly with s , i.e. resonances become narrower. This is the reverse of what happens for $q\bar{q}$ states treated as CDD poles. It seems likely that this point is at the root of the disagreement between data and the comparisons made here with $q\bar{q}$ and $q^2\bar{q}^2$ models.

Oller [27] makes a comparison with a scheme along these lines where K-matrix elements for σ and κ are taken from chiral perturbation theory. Resonances are then generated dynamically. This gives a more promising agreement with $SU(3)$ and he claims to obtain reasonable agreement with a $q\bar{q}$ nonet. However, he predicts a coupling of $f_0 \rightarrow \eta\eta$ (which is nearly the same as his $\eta_8\eta_8$) almost as large as to KK . The new BES data for $f_0(980)$ rule out that possibility, which would lead to a dramatic fall in the $f_0 \rightarrow KK$ and $\pi\pi$ signals at the $\eta\eta$ threshold.

4 Structure at the $\eta\eta'$ threshold?

An important experimental question is whether there is a further $s\bar{s}s\bar{s}$ state. This is foreign to Jaffe's nonet. GAMS have reported tentative evidence for a narrow state in $\eta\eta'$ at 1914 MeV, almost exactly the $\eta'\eta'$ threshold [28]. They claim $\Gamma(\pi^0\pi^0)/\Gamma(\eta\eta') < 0.1$. Such a state should decay easily to $\eta\eta'$. If it were narrow, as GAMS claim ($\Gamma = 90_{-50}^{+35}$ MeV), its decays to $\eta'\eta'$ would be suppressed by phase space. This is a similar situation to $f_0(980)$, which appears as a narrow cusp in $\pi\pi$, but is much more difficult to observe in KK , despite strong coupling to that channel. Barberis et al. [29] report an $\eta\eta'$ enhancement with $M = 1934 \pm 16$ MeV, $\Gamma = 141 \pm 41$ MeV, but favour quantum numbers $J^{PC} = 2^{++}$. This could well be a different well-known resonance $f_2(1920)$, seen prominently in decays to $\pi\pi$, $\omega\omega$ and $\eta\pi\pi$. They also see a threshold enhancement in $\eta'\eta'$ with $M = 2007 \pm 24$ MeV, $\Gamma = 90 \pm 43$ MeV; this could be $f_2(1920)$ ($q\bar{q} \ ^3P_0$) or its well established 3F_2 partner $f_2(2001)$, observed in all of $\pi\pi$, $\eta\eta$, $\eta\eta'$ and $f_2\eta$ with consistent mass and width [30]. The key to sorting out this situation is to get high statistics data on $\eta\eta'$ with good mass resolution.

5 Summary

There is evidence that $f_0(980)$ must have a substantial KK cloud, as predicted in Sect. 2. The basic pointer to this

conclusion is that the ratio $g^2(f_0 \rightarrow KK)/g^2(a_0 \rightarrow KK)$ is at least a factor 2 larger than can be fitted by either $q\bar{q}$ or $q^2\bar{q}^2$.

Otherwise, conclusions are negative; but it may be important to know what does not work. This negative conclusion should not be surprising for $q\bar{q}$, in view of the fact that masses are far from the usual nonet configurations such as ω , ρ , $K^*(890)$ and ϕ . Predictions for relative widths of κ and σ fail badly for both $q\bar{q}$ and Jaffe's model. Meson exchange models fare better in predicting $\pi\pi$ and $K\pi$ phase shifts. These models predict that σ , κ , f_0 and a_0 are only just bound. The large decay widths of σ and κ reflect this fact: they decay easily to lighter $\pi\pi$ and $K\pi$ systems. An approach along these lines will be considered in a separate paper.

Acknowledgements. I am grateful to Dr. F. Kleefeld, Prof. G. Rupp, Prof. E. van Beveren, and Dr. C. Hanhart for extensive discussions about formulae for branching ratios. Also to Prof. R. Jaffe for delving into 29 year-old notes for detailed derivation of amplitudes in his model.

References

1. V.E. Markushin, M. P Locher, Frascati Physics Series **15**, 229 (1999)
2. E.M. Aitala et al., Phys. Rev. Lett. **86**, 765 (2001)
3. M. Ablikim et al., Phys. Lett. B **598**, 149 (2004)
4. R.J. Jaffe, Phys. Rev. D **15**, 267 (1977)
5. D.V. Bugg, Phys. Lett. B **632**, 471 (2006)
6. F. Okiharu et al., hep-ph/0507187
7. D. Lohse, J.W. Durso, K. Holinde, J. Speth, Phys. Lett. B **234**, 235 (1990)
8. B.S. Zou, D.V. Bugg, Phys. Rev. D **50**, 591 (1993)
9. M.P. Locher, V.E. Markushin, H.Q. Zheng, Eur. Phys. J. C **4**, 317 (1998)
10. G. Janssen, B.C. Pearce, K. Holinde, J. Speth, Phys. Rev. D **52**, 2690 (1995)
11. L. Castillejo, R.H. Dalitz, F.J. Dyson, Phys. Rev. **101**, 453 (1956)
12. E. van Beveren et al., Z. Phys. C **30**, 615 (1986)
13. E. van Beveren, G. Rupp, N. Petropoulos, F. Kleefeld, AIP Conf. Proc. **660**, 353 (2003)
14. E. van Beveren, G. Rupp, Phys. Rev. Lett. **91**, 012003 (2003)
15. N.A. Törnqvist, Z. Phys. C **68**, 647 (1995)
16. M. Ablikim et al., Phys. Lett. B **607**, 243 (2005)
17. V. Baru et al., Phys. Lett. B **586**, 53 (2004)
18. D.V. Bugg, V.V. Anisovich, A.V. Sarantsev, B.S. Zou, Phys. Rev. D **50**, 4412 (1994)
19. N.N. Achasov, G.N. Shestakov, Phys. Rev. D **49**, 5779 (1994)
20. J.A. Oller, Phys. Rev. D **71**, 054030 (2005)
21. D.V. Bugg, Phys. Lett. B **572**, 1 (2003)
22. A.V. Anisovich, V.V. Anisovich, A.V. Sarantsev, Z. Phys. A **359**, 173 (1997)
23. D. Aston et al., Nucl. Phys. B **296**, 253 (1988)
24. D.V. Bugg, accompanying paper [hep-ex/0603023]
25. B. Hyams et al., Nucl. Phys. B **64**, 134 (1973)
26. C. Amsler et al., Phys. Lett. B **294**, 451 (1992)
27. J.A. Oller, Nucl. Phys. A **727**, 353 (2003)
28. D.M. Alde et al., Sov. J. Nucl. Phys. **54**, 455 (1991); Phys. Lett. B **276**, 375 (1992)
29. D. Barberis et al., Phys. Lett. B **471**, 429 (2000)
30. D.V. Bugg, Phys. Rep. **397**, 257 (2004)